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DIFFUSION PROBLEM OF A CREEPING GAS.

I. CALCULATION OF DISTRIBUTION FUNCTIONS OF A MOLECULAR GAS IN THE KNUDSEN LAYER

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A modified method of half range expansions has been used to find distribution functions of a molecular gas mixture inhomogeneous in concentration in the Knudsen layer.

The study of dilute gas flow at finite Knudsen numbers can be carried out successfully within hydrodynamics; i.e., within models of a continuous medium, taking into account; however, corrections related to the nonvanishing of the Knudsen number primarily in the boundary conditions. This approach assumes the introduction of "jumps" of macroscopic parameters, such as temperature, gas velocity, etc., at the phase boundary, and the problem reduces to more or less rigorous calculations or estimates of the corresponding coefficients.

Unfortunately, a more general and informative method is that of finding the velocity distribution function of gas molecules, including the layer immediately adjacent to the boundary — the Knudsen layer. This approach requires the solution of a quite complicated kinetic equation, which cannot be done in the general case.

The approximate methods adopted for this purpose consist of either replacing the Boltzmann equation by a model equation with its subsequent solution, or searching an approximate solution of the Boltzmann equation itself by one of the well-known methods. In our opinion, the most fruitful method is that of the half-range expansions, suggested in [1]. Its application in the original form is rendered difficult by the complexity of calculating the half-range moments of the Boltzmann collision integral, integral parentheses (see, e.g., [2]). Papers were later published [3-7], however, in which a modification of the method of [1] was suggested, making it possible to shorten the number of integral brackets requiring direct evaluation, and to simplify the system of moment equations for determining the expansion coefficients of the unknown molecular distribution functions.

This modified method of half-range expansions was used to find molecular distribution functions of a simple gas in the Knudsen layer, which in turn made it possible to accurately calculate the isothermal [6] and thermal [7] coefficients of a creeping gas.

An attempt of applying the half-range expansion method to the calculation of distribution functions for a gas mixture was undertaken in [8, 9]. The author of these papers, however, did not correctly calculate the half-range moments of the Boltzmann integral, a task involving great computational difficulty. In fact, the calculations of these expressions for a gas mixture is incomparably harder than in the case of a simple gas. Therefore, a modification similar to [3-7] is of considerable interest.

A detailed analysis of the procedure of constructing the moment equations for the kinetic Boltzmann equation was performed in [10]. It was shown that the Onsager mutuality principle makes it possible to remove the uncertainty in the choice of moments. We take this fact into account in obtaining a system of equations for determining expansion coefficients in polynomials of velocity distribution functions of components of a binary gas mixture.

In the present paper we establish a number of important relations between the half-range moments of the Boltzmann collision integral for a gas mixture, and we calculate the necessary moments. Based on that, we calculate molecular distribution functions in the Knudsen layer and

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the diffusion coefficient of a creeping binary as mixture for arbitrary values of the accommodation coefficients of the tangential momentum of gas molecules on the surface of a solid.

1. Consider the problem of slow stationary flow of a binary gas mixture in a plane-parallel channel with distance $2d$ between the plates. At constant pressure and temperature the Boltzmann equation for the mixture components is

$$f_i^{(0)} v_{iz} \frac{\partial \ln n_i}{\partial z} + f_i^{(0)} v_{ix} \frac{\partial \varphi_i}{\partial x} = \sum_j I_{ij}(\varphi_i + \varphi_j), \quad (1)$$

where φ_i ($i = 1, 2$) are small corrections to the solution of the zeroth-approximation Boltzmann equation $f_i^{(0)}$ (the Maxwell function), z and x are coordinates directed, respectively, along and across the flow, and $I_{ij}(\varphi_i + \varphi_j)$ is the linearized collision integral.

We seek φ_i in the form of expansions in polynomials in velocity ($\mathbf{c}_i = \mathbf{v}_i \sqrt{m_i/2kT}$):

$$\varphi_i(\mathbf{c}_i, x) = \frac{1}{2} \sum_{h=0}^N a_h^{(i)} P_h(\mathbf{c}_i). \quad (2)$$

Restricting ourselves to the case $N = 3$, we choose for $P_k(\mathbf{c})$ the polynomials

$$P_0 = c_{iz}, P_1 = c_{iz} \frac{c_{ix}}{|c_{ix}|}, P_2 = c_{iz} \frac{c_{ix}^2}{|c_{ix}|}, P_3 = c_{ix} c_{iz}. \quad (3)$$

Substituting expansion (2) into Eq. (1), multiplying successively by the polynomials $P_l(\mathbf{c}_i)$ and integrating over velocity, we obtain a system of linear differential equations for the coefficients $a_k^{(i)}(x)$:

$$2K_i \frac{\partial n_i}{\partial z} J_l + n_i K_i \sum_h \frac{\partial a_h}{\partial x} N_{lh} = \sum_h \left(K_i n_i a_h^{(i)} \mathcal{L}_{lh}^{(i)} + n_i n_j \sum_{j=1}^2 a_h^{(j)} M_{lh}^{(ij)} \right). \quad (4)$$

Here

$$K_i = \left(\frac{2kT}{m_i \pi^3} \right)^{1/2}; \quad J_l = \int c_{iz} P_l(\mathbf{c}_i) e^{-c_i^2} d\mathbf{c}_i;$$

$$N_{lh} = \int c_{ix} P_l(\mathbf{c}_i) P_h(\mathbf{c}_i) e^{-c_i^2} d\mathbf{c}_i;$$

$$\mathcal{L}_{lh}^{(i)} = \frac{n_i}{\pi^{3/2}} \int e^{-c_i^2 - c_i'^2} |c_i - c_i'| d\Sigma_{ii} d\mathbf{c}_i d\mathbf{c}_i' P_l(\mathbf{c}_i) [P_h(\mathbf{c}_i') + P_h(\mathbf{c}_i) - P_h(\mathbf{c}_i) - P_h(\mathbf{c}_i)]; \quad (5)$$

$$M_{lh}^{(ij)} = \frac{1}{\pi^{3/2}} \int e^{-c_1^2 - c_2^2} |\mathbf{v}_1 - \mathbf{v}_2| d\mathbf{c}_1 d\mathbf{c}_2 d\Sigma_{12} P_l(\mathbf{c}_i) [P_h(\mathbf{c}_j') - P_h(\mathbf{c}_j)],$$

where \mathbf{c}_i' are molecular velocities after collisions, and $d\Sigma_{ii}$ is the differential scattering cross section.

We note that the moments $\mathcal{L}_{lk}^{(i)}$ and $M_{lk}^{(ij)}$ vanish if the indices i and k are of different parity. The moments $\mathcal{L}_{ik}^{(i)}$, one of whose indices vanishes, also vanish. Also, $\mathcal{L}_{lk}^{(i)}$ are symmetric in the permutation $l \leftrightarrow k$, and $M_{lk}^{(ij)}$ do not change under simultaneous interchange of the upper and lower indices.

Consider the integral brackets $M_{ko}^{(ij)}$ and $M_{ko}^{(ii)}$:

$$M_{k0}^{(ij)} = \frac{1}{\pi^{3/2}} \int e^{-c_1^2 - c_2^2} |\mathbf{v}_1 - \mathbf{v}_2| d\mathbf{c}_1 d\mathbf{c}_2 d\Sigma_{12} P_k(\mathbf{c}_i) [c_{2i}' - c_{2i}], \quad (6)$$

$$M_{k0}^{(ii)} = \frac{1}{\pi^{3/2}} \int e^{-c_1^2 - c_2^2} |\mathbf{v}_1 - \mathbf{v}_2| d\mathbf{c}_1 d\mathbf{c}_2 d\Sigma_{12} P_k(\mathbf{c}_i) [c_{2i}' - c_{2i}],$$

and multiply the upper relation by $(m_j)^{1/2}$ and the second by $(m_i)^{1/2}$, and add. By the momentum conservation law

$$\sqrt{m_i}c_i' + \sqrt{m_j}c_j' = \sqrt{m_i}c_i + \sqrt{m_j}c_j,$$

and we obtain the first set of required relations

$$M_{k0}^{(ij)} \sqrt{m_j} + M_{k0}^{(ii)} \sqrt{m_i} = 0. \quad (7)$$

Other relations are obtained by using the system of equations (4) in the limiting case $\lambda/d \ll 1$ (λ is a characteristic size of the order of the molecular mean free path).

When the number of intermolecular collisions is large, the functions $a_1^{(i)}$, $a_2^{(i)}$ and the derivatives $\frac{\partial}{\partial x} a_3^{(i)}$ vanish in the bulk of the gas removed from the walls (this follows from the requirement of transition of expansion (2) to the Chapman-Enskog solution in the gas bulk), and the system of equations (4) is considerably simplified. Dividing for $\partial n_i / \partial z = 0$ (the Couette problem) the "bulk" equations with $l = 1$ and 3 by $2/\sqrt{\pi} n_i$ and n_i , respectively, and denoting the mean gas velocity in the bulk, coinciding with the velocity of each of the mixture components, by the symbol $\langle v_z \rangle$:

$$\langle v_z \rangle = -\frac{1}{n_i} \int f_i^{(0)} \varphi_i v_{iz} d\mathbf{v}_i = \frac{\pi \sqrt{\pi}}{4} K_i a_0^{(i)}, \quad (8)$$

we obtain the expression $\partial/\partial x \langle v_z \rangle$ on all left-hand sides of the four equations. The last fact makes it possible to write down the following relations:

$$\frac{K_2 \mathcal{L}_{13}^{(2)} + n_1 M_{13}^{(22)} - n_2 M_{13}^{(12)}}{K_1 \mathcal{L}_{13}^{(1)} + n_2 M_{13}^{(11)} - n_1 M_{31}^{(12)}} = \frac{K_2 \mathcal{L}_{13}^{(2)} + n_1 \frac{2}{\sqrt{\pi}} M_{33}^{(22)} - n_2 M_{13}^{(12)}}{K_1 \mathcal{L}_{13}^{(2)} + n_2 M_{13}^{(11)} - n_1 \frac{2}{\sqrt{\pi}} M_{33}^{(12)}} = \frac{K_2 \mathcal{L}_{13}^{(2)} + n_1 M_{13}^{(22)} - n_2 \frac{2}{\sqrt{\pi}} M_{33}^{(12)}}{K_1 \mathcal{L}_{13}^{(1)} + n_2 \frac{2}{\sqrt{\pi}} M_{33}^{(11)} - n_1 M_{31}^{(12)}}, \quad (9)$$

being a consequence of the identity of the right-hand sides of these equations.

Keeping in mind that $M \sim \sigma_{12}^2$, $\mathcal{L}^{(2)} \sim \sigma_2^2$, $\mathcal{L}^{(1)} \sim \sigma_1^2$ ($\sigma_{12} = 1/2(\sigma_1 + \sigma_2)$, σ_1 and σ_2 are molecular diameters of the first and second sort, considered as impenetrable spheres), we obtain that to satisfy (9) it is necessary and sufficient that the following equalities hold

$$\frac{M_{13}^{(11)}}{M_{33}^{(11)}} = \frac{M_{13}^{(22)}}{M_{33}^{(22)}} = \frac{M_{31}^{(12)}}{M_{33}^{(12)}} = \frac{M_{13}^{(12)}}{M_{33}^{(12)}} = \frac{2}{\sqrt{\pi}}. \quad (10)$$

For $\partial/\partial x \langle v_z \rangle = 0$, $\partial n_i / \partial z \neq 0$ (the problem of gas diffusion in a plane-parallel channel) we have from the equations of system (4) with $l = 0.2$, with account of relations (7)

$$\frac{M_{00}^{(11)}}{M_{02}^{(11)}} = \frac{M_{00}^{(12)}}{M_{02}^{(11)}} = \frac{M_{00}^{(12)}}{M_{02}^{(12)}} = \frac{M_{00}^{(22)}}{M_{02}^{(22)}} = \sqrt{\pi}. \quad (11)$$

It is interesting to point out that the accuracy with which the relationships (7), (10), (11) is satisfied varies. Equalities (7) follow directly from the momentum conservation law for colliding molecules and are, therefore, exact. A direct calculation of the moments $M_{00}^{(11)}$, $M_{00}^{(12)}$, $M_{00}^{(22)}$ performed in [11] verifies this result. In deriving equalities (10) and (11), expansion (2) was used extensively, i.e., the shape of approximate distribution functions near the walls, as well as the requirement of continuous transition of this function to the Chapman-Enskog distribution in the gas bulk. Hence, obviously follows their approximate nature. A direct calculation of the moments $M_{02}^{(11)}$, $M_{13}^{(11)}$, $M_{13}^{(12)}$, performed by us within the model of solid impenetrable spheres for an arbitrary relation of molecular masses in the mixture, made it possible to estimate the accuracy with which equalities (10) and (11) are satisfied. The following expressions were obtained:

$$M_{02}^{(11)} = I_0 \frac{\mu_2 \sqrt{\mu_2}}{2} \left[\frac{1}{4} \left(\frac{\mu_1}{\mu_2} \right)^{3/2} - \frac{\mu_1}{\mu_2} \arcsin \sqrt{\mu_2} + \left(2 - \frac{1}{8} \cdot \frac{1}{\mu_2} \right) \left(\sqrt{\frac{\mu_1}{\mu_2}} + \frac{1}{\mu_2} \arcsin \sqrt{\mu_2} \right) \right], \quad (12)$$

$$M_{13}^{(11)} = M_{02}^{(11)} - I_0 \frac{\mu_1 \sqrt{\mu_2}}{2} \left[\frac{\mu_1}{\mu_2} \arcsin \sqrt{\mu_2} - \left(\mu_2 + \frac{1}{2} \right) \left(\sqrt{\frac{\mu_1}{\mu_2}} + \frac{1}{\mu_2} \arcsin \sqrt{\mu_2} \right) \right], \quad (13)$$

$$M_{13}^{(12)} = I_0 \mu_1 \sqrt{\mu_2} \left[\left(\frac{3}{16} \frac{1}{\mu_2} - \frac{1}{4} - \frac{\mu_2}{2} \right) \left(\sqrt{\frac{\mu_1}{\mu_2}} + \frac{1}{\mu_2} \arcsin \sqrt{\mu_2} \right) - \frac{3}{8} \left(\frac{\mu_1}{\mu_2} \right)^{3/2} \right], \quad (14)$$

where

$$I_0 = - \left(\frac{2kT}{\mu_1 \mu_2} \right)^{1/2} \sigma_{12}^2; \quad \mu_i = \frac{m_i}{m_1 + m_2}.$$

Figure 1 shows curves of the dependences of $R_1 = M_{00}^{(11)} / \sqrt{\pi} M_{02}^{(11)}$, $R_2 = 2M_{33}^{(11)} / \sqrt{\pi} M_{13}^{(11)}$, $R_3 = 2M_{33}^{(12)} / \sqrt{\pi} M_{13}^{(12)}$ on μ_2 (the expressions for $M_{00}^{(11)}$, $M_{33}^{(12)}$, and $M_{33}^{(11)}$ were taken from [11]). It is seen that for decreasing μ_2 (or μ_1 for the other equalities, obtained by the simple replacement $\mu_1 \leftrightarrow \mu_2$ by means of (7)), equalities (10) and (11) become all the more accurate. The maximum deviation of the relations considered from unity does not exceed 20%. This value obviously estimates the accuracy of approximation (2) for the type of problems considered.

These considerations show that the use of expressions (12), (13), (14) for moments of the collision integral within the approximate theory provides excessive accuracy which, on the one hand, complicates significantly the mathematical solution of the problem, and on the other, can lead (and often does) to physically absurd results (a simpler expression, obtained by means of relations (10), and (11), must be used).

The method suggested allows accurate estimates of the moments of the Boltzmann collision integral even for molecules with arbitrary intermolecular interactions.

It thus remained to calculate the four moments $M_{11}^{(11)}$, $M_{11}^{(12)}$, $M_{22}^{(11)}$, $M_{22}^{(12)}$. We have performed a corresponding calculation for the rigid sphere model, in which case we could substantially simplify the procedure of calculation discussed in [2]. The following expressions were obtained for these moments:

$$\begin{aligned} M_{11}^{(12)} = & -I_0 \frac{\sqrt{\pi}}{4} \sqrt{\mu_1 \mu_2} \left\{ \frac{16}{3} + \frac{3}{\sqrt{\mu_1 \mu_2}} - \frac{2}{3} \frac{1}{(\mu_1 \mu_2)^{3/2}} + \right. \\ & \left. + (\sqrt{\mu_1} + \sqrt{\mu_2}) \left[\frac{2}{3} \frac{1}{(\mu_1 \mu_2)^{3/2}} - \frac{2}{3} \frac{1}{\mu_1 \mu_2} - \frac{3}{\sqrt{\mu_1 \mu_2}} \right] - \frac{|V_{\mu_1} - V_{\mu_2}|^3}{\sqrt{\mu_1 \mu_2}} \right\}, \\ M_{11}^{(11)} = & I_0 \sqrt{\pi} \left(\mu_1 \sqrt{\mu_2} + \frac{4}{3} \mu_2 \right), \\ M_{22}^{(11)} = & I_0 \frac{\sqrt{\pi}}{2} \left[\frac{8}{3} \mu_1 \mu_2 + \frac{8}{5} \mu_2^2 - \mu_1 \mu_2^{3/2} \right], \\ M_{22}^{(12)} = & -I_0 \frac{\sqrt{\pi}}{4} \sqrt{\mu_1 \mu_2} \left\{ \frac{32}{15} \sqrt{\mu_1 \mu_2} + \frac{5}{6} \frac{1}{\mu_1 \mu_2} - \frac{1}{5} \frac{1}{(\mu_1 \mu_2)^2} + \right. \\ & \left. + (\sqrt{\mu_1} + \sqrt{\mu_2}) \left(\frac{9}{8} - \frac{3}{8} \sqrt{\mu_1 \mu_2} + \frac{27}{8} \mu_1 \mu_2 - \frac{3}{8} (\mu_1 \mu_2)^{3/2} - \right. \right. \\ & \left. \left. - \frac{15}{4} (\mu_1 \mu_2)^2 + \frac{4}{3} \frac{1}{\sqrt{\mu_1 \mu_2}} - \frac{77}{60} \frac{1}{\mu_1 \mu_2} - \frac{1}{5} \frac{1}{(\mu_1 \mu_2)^{3/2}} + \right. \right. \\ & \left. \left. + \frac{1}{5} \frac{1}{(\mu_1 \mu_2)^2} \right) + |V_{\mu_1} - V_{\mu_2}| \left[\frac{11}{8} - \frac{11}{8} \sqrt{\mu_1 \mu_2} - \frac{27}{8} \mu_1 \mu_2 - \right. \right. \\ & \left. \left. - \frac{45}{8} (\mu_1 \mu_2)^{3/2} - \frac{1}{8} \frac{1}{\mu_1 \mu_2} - \frac{1}{40} \frac{1}{\sqrt{\mu_1 \mu_2}} + \frac{21}{40} \frac{1}{(\mu_1 \mu_2)^{3/2}} \right] \right\}. \end{aligned} \quad (15)$$

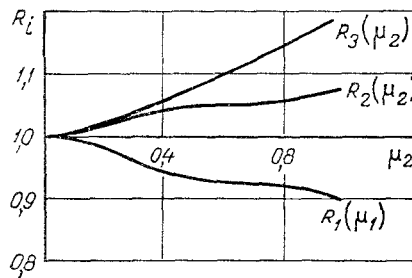


Fig. 1. Ratio of approximately calculated moments to the accurate moments as a function of $m_2 / (m_1 + m_2)$.

The moments $M_{11}^{(22)}$ and $M_{22}^{(22)}$ are obtained from the moments $M_{11}^{(11)}$ and $M_{22}^{(11)}$ by replacing $\mu_1 \rightarrow \mu_2$.

2. We turn now to direct solution of the problem of diffusion creeping. The system of equations (4) must be supplemented by boundary conditions, taken to be Maxwellian: in molecular collisions with the surface of the channel walls a fraction ε_i of molecules of the i -th sort is reflected by diffusion, and $1 - \varepsilon_i$ by mirror reflection, which gives

$$a_{1,3}^{(i)}(d) = \frac{\varepsilon_i}{2 - \varepsilon_i} a_{0,2}^{(i)}(d). \quad (16)$$

In solving the system of equations (4), we restrict ourselves to the case of a dilute binary gas mixture; i.e., we assume a small number of molecules of the second sort $n_2/n \ll 1$ with collision frequency $K_2 \sigma_{12}^2 n_2 \ll K_1 \sigma_{11}^2 n_1$, and we seek $a_k^{(i)}(x)$ in the form

$$a_k^{(1)} = a_{k_0}^{(1)} + \frac{n_2}{n} \xi_k. \quad (17)$$

We note that the boundary conditions for $a_{k_0}^{(i)}$ and ξ_k are of the form (16). We substitute expansion (17) into system (4). In the zeroth approximation in n_2/n we have a set of equations for $a_{k_0}^{(i)}$, which for boundary conditions (16) satisfies the zeroth-order solution. Therefore, in what follows one can put $a_{k_0}^{(i)} \equiv 0$ without restricting the generality of approach.

Retaining terms linear in the ratio n_2/n , we obtain a system of equations determining $a_k^{(2)}(x)$:

$$2K_2 \frac{\partial \ln n_2}{\partial z} J_l + \sum_{k=0}^3 \left[\left(K_2 N_{lk} \frac{\partial}{\partial x} - M_{lk}^{(22)} \right) a_k^{(2)}(x) \right] = 0 \quad (18)$$

and a system of equations for $\xi_k(x)$:

$$-2K_1 \frac{\partial \ln n_2}{\partial z} J_l + \sum_{k=0}^3 \left[\left(K_1 N_{lk} \frac{\partial}{\partial x} - \mathcal{L}_{lk}^{(1)} \right) \xi_k - n_1 a_k^{(2)} M_{lk}^{(12)} \right] = 0. \quad (19)$$

The solution of system (18) is found by standard procedures. To determine the function ξ_k it is convenient to use relations (7) and (10) between moments of the Boltzmann collision integral, which makes it possible to integrate quite easily the two equations of system (19).

The general result can be represented in the form

$$a_k^{(2)}(x) = \delta_{0k} E + \begin{cases} \sum_{r=1}^2 C_r \gamma_{kr} \operatorname{ch} \alpha_r x, & k = 0, 2, \\ \sum_{r=1}^2 C_r \gamma_{kr} \operatorname{sh} \alpha_r x, & k = 1, 3, \end{cases} \quad (20)$$

$$\xi_k = A\delta_{0k} + \begin{cases} \beta_k \operatorname{ch} \alpha_0 x + \sum_r C_r \Delta_{kr} \operatorname{ch} \alpha_r x, & k = 0, 2, \\ \beta_k \operatorname{sh} \alpha_0 x + \sum_r C_r \Delta_{kr} \operatorname{sh} \alpha_r x, & k = 1, 3. \end{cases} \quad (21)$$

Here δ_{kr} is the Kronecker symbol. The eigenvalues α_r , the coefficients E , γ_{kr} , Δ_{kr} , and the integration constants A , C_r , β_k are given in the Appendix.

It is now easy to write down the mean bulk velocity of the binary gas mixture

$$\langle v_z \rangle = \sum_{i=1}^2 \frac{\rho_i}{\rho} \sqrt{\frac{kT}{8m_i}} \left(\alpha_0^{(i)} + \frac{1}{\sqrt{\pi}} \alpha_2^{(i)} \right),$$

where ρ_i is the partial density of component i , and i , $\rho = \rho_1 + \rho_2$. Substitution of expressions (20) and (21) gives

$$\langle v_z \rangle = D_{12} \frac{\rho_1}{\rho_2} \frac{\partial \ln n_1}{\partial z} \left[\frac{m_2}{m_1} - \sqrt{\frac{m_2}{m_1}} \frac{A}{E} + \sum_{q=0}^2 \Phi_q \operatorname{ch} \alpha_q x \right]. \quad (22)$$

We provide here the mutual diffusion coefficient of the mixture components according to the first approximation of the Chapman-Enskog method [11]

$$D_{12} = \frac{1}{2} \frac{kT}{n \sqrt{m_1 m_2}} \frac{1}{M_{00}^{(12)}}.$$

The symbol Φ_q denotes the functions describing the velocity profile of the gas in the Knudsen layer:

$$\begin{aligned} \Phi_0 &= -\sqrt{\frac{m_2}{m_1}} \frac{\beta_0}{2E}, \\ \Phi_r &= -\sqrt{\frac{m_2}{m_1}} \frac{C_r}{E} \left(\Delta_{0r} + \sqrt{\frac{m_2}{m_1}} \left(\gamma_{0r} + \frac{1}{\sqrt{\pi}} \gamma_{2r} \right) \right), \quad r = 1, 2. \end{aligned}$$

For $\alpha_j d \gg 1$ the last term in (22) tends to zero in the gas bulk, and the velocity of gas diffusion slip can be determined as

$$u_{DS} = D_{12} \frac{\rho_1}{\rho} \frac{\partial \ln n_1}{\partial z} \left[\frac{m_2}{m_1} - \sqrt{\frac{m_2}{m_1}} \frac{A}{E} \right] = K_{DS} D_{12} \frac{\partial \ln n_1}{\partial z}, \quad (23)$$

where K_{DS} is the coefficient of diffusion slip.

For $\varepsilon_i \rightarrow 0$ Eq. (23) acquires the quite simple form:

$$u_{DS} = D_{12} \frac{\partial \ln n_1}{\partial z} \left[\frac{m_2}{m_1} - \sqrt{\frac{m_2}{m_1}} \frac{\varepsilon_2}{\varepsilon_1} \right]$$

and for $\varepsilon_1 = \varepsilon_2$ coincides with that obtained in [12] within the approximate Maxwell method. For arbitrary values of ε_i the diffusion slip coefficient can be represented in the form

$$\begin{aligned} K_{DS} &= \frac{m_2}{m_1} - \sqrt{\frac{m_2}{m_1}} \frac{2 - \varepsilon_1}{\varepsilon_1} \frac{\varepsilon_2}{2 - \varepsilon_2} \frac{1}{1 - \frac{\varepsilon_1}{2 - \varepsilon_1} \frac{\beta_2}{\beta_3}} \times \\ &\times \left\{ F_0 + \frac{\varepsilon_1}{2 - \varepsilon_1} H_1 + \left(\frac{\varepsilon_1}{2 - \varepsilon_1} \right)^2 H_2 + \frac{\varepsilon_2}{2 - \varepsilon_2} \left[Q_0 + \frac{\varepsilon_1}{2 - \varepsilon_1} Q_1 + \right. \right. \\ &\left. \left. + \left(\frac{\varepsilon_1}{2 - \varepsilon_1} \right)^2 Q_2 \right] \right\} \left\{ F_0 + \frac{\varepsilon_2}{2 - \varepsilon_2} F_1 + \left(\frac{\varepsilon_2}{2 - \varepsilon_2} \right)^2 F_2 \right\}^{-1}, \end{aligned}$$

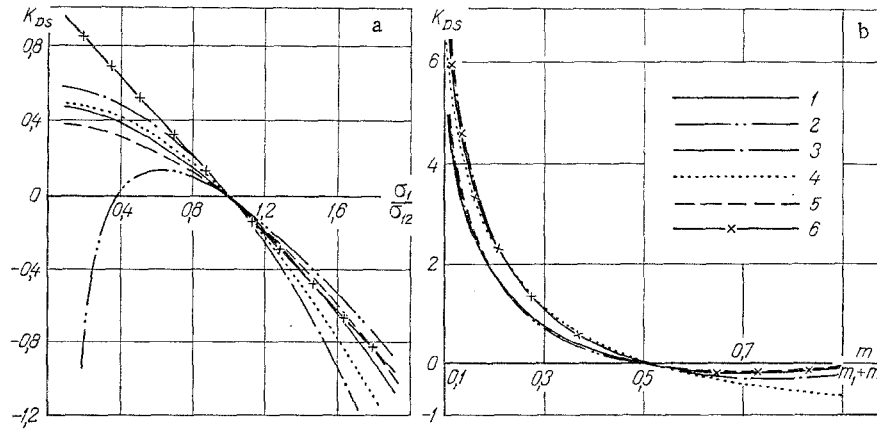


Fig. 2. Diffusion slip coefficient as a function of σ_1/σ_{12} for $m_1 = m_2$ (a) and as a function of $m_1/(m_1 + m_2)$ for $\sigma_1 = \sigma_2$ (b): 1) results of the present work, 2) [13]; 3) [14]; 4) [15]; 5) [16]; 6) [17].

where F_i , H_i , and Q_i , are algebraic functions of the parameters γ_{ij} and Δ_{ij} .

A calculation shows that the quantity K_{DS} depends strongly on the relation between the molecular diameters of the mixture components.

Figure 2 shows the dependence of the diffusion slip coefficient on the molecular diameter (for $m_1 = m_2$) and on the molecular mass (for $\sigma_1 = \sigma_2$), obtained in the present study and in [13-17]. It is seen from the figure that all the theoretical calculations for $\sigma_1 \approx \sigma_2$ are in satisfactory agreement between them and with experiment [17]. A significant deviation between the results of [13] and the remaining calculations is obtained for $\sigma_1 \ll \sigma_2$.

All the calculations given are based on one or another approximation, therefore the problem of the source of behavior of the curve of K_{DS} in the region $\sigma_1 \ll \sigma_2$ remains open. Unfortunately, the experimental data [17] also do not allow a clear preference of one theory or another, since they were obtained for values $\sigma_1 \approx \sigma_2$.

The necessity thus exists of experimental measurements of the diffusion slip velocity for $\sigma_1 \ll \sigma_2$, as well as calculations by the proposed scheme (or by methods of [13-16]), in which a large number of terms in the distribution function expansion. Such calculations will be performed shortly by the present authors.

APPENDIX

We provide the explicit shapes of the coefficients appearing in expressions (21) and (20),

$$E = \frac{\pi \sqrt{\pi}}{M_{00}^{(12)}} \frac{K_2}{n_1} \frac{\partial \ln n_2}{\partial z};$$

$$\alpha_{1,2} = \frac{2\pi n}{4-\pi} \left(\frac{m_2}{kT}\right)^{1/2} \sqrt{W + M_{22}^{(22)} S \pm \sqrt{(W - M_{22}^{(22)} S)^2 + \frac{4}{\pi} M_{00}^{(22)} W S}};$$

$$\gamma_{1j} = \frac{\pi-4}{4} \sqrt{\pi} \frac{\alpha_j}{Sn} K_2 \gamma_{2j}; \quad \gamma_{2j} = -\frac{\sqrt{\pi}}{2} \left(1 + \frac{1}{\Delta_j}\right); \quad \gamma_{0j} = 1;$$

$$\gamma_{3j} = \frac{\pi}{2n} K_2 \alpha_j \left[\frac{4-\pi}{\pi} \frac{\gamma_{2j}}{S} - \frac{\sqrt{\pi}}{2\Delta_j M_{33}^{(22)}} \right]; \quad S = M_{11}^{(22)} - \frac{4}{\pi} M_{33}^{(22)};$$

$$\Delta_j = 1 - \frac{\alpha_j^2 K_2^2 \pi^3}{8n^2 M_{00}^{(22)} M_{33}^{(22)}}; \quad W = \left(\frac{4-\pi}{\pi}\right)^2 M_{00}^{(22)} M_{33}^{(22)};$$

$$\alpha_0 = \frac{4}{\pi(4-\pi)} \sqrt{2\sqrt{\pi} \mathcal{L}_{22}^{(1)} \left(\frac{\sqrt{\pi}}{2} \mathcal{L}_{11}^{(1)} - \mathcal{L}_{13}^{(1)}\right)}; \quad \beta_1 = -\frac{\sqrt{\pi}}{2} \beta_3;$$

$$\beta_2 = -\frac{\sqrt{\pi}}{2} \beta_0 = -\frac{\beta_3}{\alpha_0} \frac{4\sqrt{\pi}}{\pi(4-\pi)} \left(\mathcal{L}_{13}^{(1)} - \frac{\sqrt{\pi}}{2} \mathcal{L}_{11}^{(1)}\right);$$

$$\Delta_{2j} = [g_1 + g_2 \gamma_{2j} + g_0 \alpha_j (d_1 \gamma_{1j} + \gamma_{3j})] / \left(\frac{\alpha_j^2}{\alpha_0^2} - 1 \right);$$

$$\Delta_{3j} = [d_1 \gamma_{1j} + \gamma_{3j} + d_0 \alpha_j (g_1 + g_2 \gamma_{2j})] / \left(\frac{\alpha_j^2}{\alpha_0^2} - 1 \right);$$

$$\Delta_{0j} = b_1 + \frac{2}{\sqrt{\pi}} (b_1 \gamma_{2j} - \Delta_{2j}); \quad \Delta_{1j} = \left[\gamma_{1j} + \frac{\sqrt{\pi}}{2} (\gamma_{3j} + \Delta_{3j}) \right];$$

$$d_0 = \frac{\beta_2}{\beta_3} \alpha_0; \quad d_1 = \frac{2}{\sqrt{\pi}} + \frac{M_{11}^{(12)} - \frac{2}{\sqrt{\pi}} M_{31}^{(12)}}{\mathcal{L}_{13}^{(1)} - \frac{\sqrt{\pi}}{2} \mathcal{L}_{11}^{(1)}} \frac{n}{K_1};$$

$$g_0 = \frac{\pi(4-\pi)}{8\mathcal{L}_{22}^{(1)}}; \quad g_1 = \frac{M_{02}^{(11)} - \frac{\sqrt{\pi}}{2} M_{00}^{(12)}}{\mathcal{L}_{22}^{(1)}} \frac{n}{K_1};$$

$$g_2 = \frac{M_{22}^{(12)} - \frac{\sqrt{\pi}}{2} M_{02}^{(12)}}{\mathcal{L}_{22}^{(1)}} \frac{n}{K_1};$$

$$b_1 = \sqrt{\frac{m_1}{m_2}} \left(M_{31}^{(12)} - \sqrt{\frac{m_2}{m_1}} \frac{K_2}{n} \mathcal{L}_{13}^{(1)} \right) \frac{1}{M_{13}^{(22)}};$$

$$A = \sum_{j=1}^2 \frac{C_j}{D_0} \left\{ \left(\sqrt{\pi} \operatorname{sh} \alpha_0 d - \frac{\beta_2}{\beta_3} \frac{4}{\sqrt{\pi}} \frac{\varepsilon_1}{2-\varepsilon_1} \operatorname{ch} \alpha_0 d \right) \times \right.$$

$$\times \left(\Delta_{3j} \operatorname{sh} \alpha_j d - \frac{\varepsilon_1}{2-\varepsilon_1} \Delta_{2j} \operatorname{ch} \alpha_j d \right) - 2 \left(\operatorname{sh} \alpha_0 d - \frac{\varepsilon_1}{2-\varepsilon_1} \frac{\beta_2}{\beta_3} \operatorname{ch} \alpha_0 d \right)$$

$$\times \left(\Delta_{1j} \operatorname{sh} \alpha_j d + \frac{\varepsilon_1}{2-\varepsilon_1} \Delta_{0j} \operatorname{ch} \alpha_j d \right) \left. \right\};$$

$$D_0 = 2 \frac{\varepsilon_1}{2-\varepsilon_1} \left(\operatorname{sh} \alpha_0 d - \frac{\varepsilon_1}{2-\varepsilon_1} \frac{\beta_2}{\beta_3} \operatorname{ch} \alpha_0 d \right);$$

$$\beta_3 = -2 \frac{\varepsilon_1}{2-\varepsilon_1} \sum_{j=1}^2 C_j \left(\Delta_{3j} \operatorname{sh} \alpha_j d - \frac{\varepsilon_1}{2-\varepsilon_1} \Delta_{2j} \operatorname{ch} \alpha_j d \right) / D_0;$$

$$C_1 = -\frac{\varepsilon_2}{2-\varepsilon_2} \frac{E}{D} \left(\gamma_{32} \operatorname{sh} \alpha_2 d - \frac{\varepsilon_2}{2-\varepsilon_2} \gamma_{22} \operatorname{ch} \alpha_2 d \right);$$

$$C_2 = \frac{\varepsilon_2}{2-\varepsilon_2} \frac{E}{D} \left(\gamma_{31} \operatorname{sh} \alpha_1 d - \frac{\varepsilon_2}{2-\varepsilon_2} \gamma_{21} \operatorname{ch} \alpha_2 d \right);$$

$$D = \left(\gamma_{11} \operatorname{sh} \alpha_1 d - \frac{\varepsilon_2}{2-\varepsilon_2} \operatorname{ch} \alpha_1 d \right) \left(\gamma_{32} \operatorname{sh} \alpha_2 d - \right.$$

$$\left. - \frac{\varepsilon_2}{2-\varepsilon_2} \gamma_{22} \operatorname{ch} \alpha_2 d \right) - \left(\gamma_{12} \operatorname{sh} \alpha_2 d - \frac{\varepsilon_2}{2-\varepsilon_2} \operatorname{ch} \alpha_2 d \right) \times$$

$$\times \left(\gamma_{31} \operatorname{sh} \alpha_1 d - \frac{\varepsilon_2}{2-\varepsilon_2} \gamma_{21} \operatorname{ch} \alpha_1 d \right).$$

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INSTABILITY OF THE SELF-SIMILAR FRONT OF A PHASE TRANSITION

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The stability of self-similar diffusional processes with respect to small disturbances of plane, cylindrical, and spherical phase interfaces is investigated.

Self-similar processes of diffusion and heat conduction accompanying phase or chemical transformations are very common both in nature and in engineering. Many actual processes of vaporization, sublimation and condensation, dissolution, melting and crystallization, ablation, combustion, etc., enter precisely into self-similar asymptotic forms over a certain time interval dependent on the specifics of the initial conditions. In many cases, however, such an asymptotic stage of the process proves to be unstable, and the front of the phase interface or chemical reaction is considerably distorted. Two main forms of disruption of stability are possible in this case. Sometimes with weak "supercriticality," i.e., a small departure from the surface of neutral stability in parametric space in the region of instability, a regular periodic cellular structure appears at the front, the amplitude of which grows monotonically from zero with an increase in supercriticality. Sometimes upon a transition through the indicated surface disturbances of finite amplitude develop immediately at the front: dendrites appear which, losing stability in turn, form peculiar branched dendritic structures. Both these forms have been observed, e.g., in crystallization from melts and solutions [1]. A well-known example of the formation of dendrites is the appearance of hoarfrost or frost patterns on glasses upon the sudden cooling of air, when it becomes supersaturated with water vapor.

The questions of the conditions and the form of the disruption of stability are important in a scientific and an applied respect, since the onset of instability can radically alter

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